# Wilcoxon Signed-Rank Test — Exam Reference Card

n = 5 to 50 · All Alpha Levels · Formulas · Decision Rules · Worked Example

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| REJECT $H_0$<br>$W \le W_{critical}$ | $W = \min(W^+, W^-)$<br>$W_{critical}$ from table below | FAIL TO REJECT $H_0$<br>$W > W_{critical}$ |
|---|---|---|

## 6-Step Procedure

| | | |
|---|---|---|
| 1 | **Calculate differences** | $D_i = X_i - Y_i$ for each pair. Drop pairs where $D_i = 0$. Remaining count is effective n. |
| 2 | **Rank absolute differences** | Rank $\|D_i\|$ from 1 (smallest) to n (largest). Assign midranks to ties: e.g., tied 3rd and 4th both receive rank 3.5. |
| 3 | **Compute $W^+$ and $W^-$** | $W^+$ = sum of ranks for all $D_i > 0$. $W^-$ = sum of ranks for all $D_i < 0$. |
| 4 | **Compute W and verify** | $W = \min(W^+, W^-)$. Check: $W^+ + W^-$ must equal $n(n+1)/2$. |
| 5 | **Choose $\alpha$ and tail direction** | Default: two-tailed $\alpha = 0.05$. One-tailed only when direction was pre-specified before data collection. |
| 6 | **Look up and decide** | Find row n in table below. Read $W_{critical}$. If $W \le W_{critical}$ → reject $H_0$ (significant). If $W > W_{critical}$ → fail to reject $H_0$. |

## Key Formulas & Symbols

| | |
|---|---|
| $D_i = X_i - Y_i$ | Paired difference (drop if $D_i = 0$) |
| $W^+ + W^- = n(n+1)/2$ | Verification check — always confirm this before looking up the table |
| $W = \min(W^+, W^-)$ | Test statistic — the smaller of the two rank sums |
| $\mu_W = n(n+1) / 4$ | Mean of W under $H_0$ — used in normal approximation ($n > 20$) |
| $\sigma_W = \sqrt{[\, n(n+1)(2n+1) / 24 \,]}$ | Standard error of W — denominator of Z approximation |
| $Z = (W - \mu_W) / \sigma_W$ | Z-score for $n > 20$: compare $\|Z\|$ to 1.960 ($\alpha$=0.05) or 2.576 ($\alpha$=0.01) two-tailed |
| $\sigma_{W,corr} = \sqrt{[\, n(n+1)(2n+1)/24 - \Sigma(t_j^3 - t_j)/48 \,]}$ | Tie-corrected standard error: $t_j$ = count of ties in tie group j |

## Worked Example (n = 8, Two-Tailed $\alpha$ = 0.05)

**Scenario:** 8 patients have anxiety scores measured before and after CBT therapy. Data cannot be assumed normal (small sample). Using Wilcoxon signed-rank test.

| Patient | Before (X) | After (Y) | D = X–Y | \|D\| | Rank | Signed Rank |
|---|---|---|---|---|---|---|
| 1 | 14 | 13 | +1 | 1 | 1 | +1 |
| 2 | 12 | 10 | +2 | 2 | 2 | +2 |
| 3 | 16 | 13 | +3 | 3 | 3 | +3 |
| 4 | 11 | 15 | −4 | 4 | 4 | −4 ■ |
| 5 | 15 | 10 | +5 | 5 | 5 | +5 |
| 6 | 17 | 11 | +6 | 6 | 6 | +6 |
| 7 | 13 | 6 | +7 | 7 | 7 | +7 |
| 8 | 18 | 10 | +8 | 8 | 8 | +8 |

| | | | | |
|---|---|---|---|---|
| $W^+$ = 1+2+3+5+6+7+8 = **32** | $W^-$ = 4 = **4** | **W = min(32, 4) = 4** | $W_{critical}$ (n=8, α=0.05, 2T) = **3** | **4 > 3 → Fail to reject $H_0$** |

Verify: $W^+ + W^- = 32 + 4 = 36 = 8×9/2 = 36$ ✓ ■ Patient 4's score worsened after therapy — one negative rank is not enough to flip the result when 7 out of 8 patients improved.

# Wilcoxon Signed-Rank Test — Complete Critical Value Table

Both Two-Tailed and One-Tailed Headers Shown · Reject H■ if W ≤ W_critical · n = 5 to 50

statisticsfundamentals.com/tables/wilcoxon-signed-rank-table/

## Complete Reference Table — All Alpha Levels (n = 5 to 50)

The highlighted column (Two-Tailed α = 0.05 / One-Tailed α = 0.025) is the standard significance level for most academic research. Milestone rows at n = 10, 20, 30, 40, 50 are marked with a bold rule.

| n | 2T α=0.10<br>1T α=0.05 | 2T α=0.05 ★<br>1T α=0.025 | 2T α=0.02<br>1T α=0.01 | n | 2T α=0.10<br>1T α=0.05 | 2T α=0.05 ★<br>1T α=0.025 | 2T α=0.02<br>1T α=0.01 |
|---|---|---|---|---|---|---|---|
| 5 | 0 | — | — | 28 | — 130 | 116 | 101 |
| 6 | 2 | 0 | — | 29 | 140 | 126 | 110 |
| 7 | 3 | 2 | 0 | 30 | — 151 | 137 | 120 |
| 8 | 5 | 3 | 1 | 31 | 163 | 147 | 130 |
| 9 | 8 | 5 | 3 | 32 | 1 175 | 159 | 140 |
| 10 | 10 | 8 | 5 | 33 | 187 | 170 | 151 |
| 11 | 13 | 10 | 7 | 34 | 5 200 | 182 | 162 |
| 12 | 17 | 13 | 9 | 35 | 213 | 195 | 173 |
| 13 | 21 | 17 | 12 | 36 | 9 227 | 208 | 185 |
| 14 | 25 | 21 | 15 | 37 | 241 | 221 | 198 |
| 15 | 30 | 25 | 19 | 38 | 15 256 | 235 | 211 |
| 16 | 35 | 29 | 23 | 39 | 271 | 249 | 224 |
| 17 | 41 | 34 | 27 | 40 | 23 286 | 264 | 238 |
| 18 | 47 | 40 | 32 | 41 | 302 | 279 | 252 |
| 19 | 53 | 46 | 37 | 42 | 32 319 | 294 | 266 |
| 20 | 60 | 52 | 43 | 43 | 336 | 310 | 281 |
| 21 | 67 | 58 | 49 | 44 | 42 353 | 327 | 296 |
| 22 | 75 | 65 | 55 | 45 | 371 | 343 | 312 |
| 23 | 83 | 73 | 62 | 46 | 54 389 | 361 | 328 |
| 24 | 91 | 81 | 69 | 47 | 407 | 378 | 345 |
| 25 | 100 | 89 | 76 | 48 | 68 426 | 396 | 362 |
| 26 | 110 | 98 | 84 | 49 | 446 | 415 | 379 |
| 27 | 119 | 107 | 92 | 50 | 83 466 | 434 | 397 |

*★ Two-tailed α = 0.05 is identical to one-tailed α = 0.025. These columns share the same critical values.*

### Alpha Level Summary & Use Cases

| Two-Tailed α | One-Tailed α | Stringency | Minimum n | Typical Use Case |
|---|---|---|---|---|
| 0.10 | 0.05 | Liberal | n = 5 | Pilot / exploratory studies; high power priority |
| **0.05** | **0.025** | **Standard** | **n = 6** | Most academic, clinical & behavioural research (default) |
| 0.02 | 0.01 | Moderate | n = 7 | Medical studies, safety-critical decisions |
| 0.01 | 0.005 | Stringent | n = 8 | Regulatory submissions, pre-registered replication studies |

### Wilcoxon Signed-Rank Test vs Paired t-Test — Quick Comparison

| Feature | Wilcoxon Signed-Rank Test | Paired Student's t-Test |
|---|---|---|
| **Data type** | Ordinal, interval, or ratio | Continuous interval or ratio |
| **Distribution** | Distribution-free (non-parametric) | Differences must be ~Normal |
| **Outlier sensitivity** | Robust — ranks limit extreme impact | Sensitive — outliers distort mean |
| **Comparison basis** | Median of paired differences | Mean of paired differences |
| **Best for** | Small n, non-normal, ordinal data | Large n, verified normality |
| **Relative power** | ~95.5% of t-test on normal data | 100% on normal data |

### Primary References


1. Wilcoxon, F. (1945). Individual comparisons by ranking methods. *Biometrics Bulletin, 1*(6), 80–83. doi:10.2307/3001968
2. Conover, W. J. (1999). *Practical Nonparametric Statistics* (3rd ed.). Wiley. [Table A.1, critical values of the Wilcoxon signed-rank statistic]
3. NIST/SEMATECH (2012). e-Handbook of Statistical Methods — Wilcoxon Signed-Rank Test. itl.nist.gov/div898/handbook/prc/section2/prc262.htm
4. Penn State STAT 415. Non-parametric Tests — Wilcoxon Signed-Rank. online.stat.psu.edu/stat415/lesson/20/20.2
5. Lehmann, E. L. & D'Abrera, H. J. M. (2006). *Nonparametrics: Statistical Methods Based on Ranks*. Springer. [ARE = 3/π ≈ 0.955 relative efficiency result]
6. Harvard T.H. Chan School of Public Health — Biostatistics Dept. (2021). Nonparametric Methods course notes. hsph.harvard.edu