

# Tukey's Q Table — Complete Reference Card

Studentized Range Critical Values |  $\alpha = 0.05$  &  $\alpha = 0.01$  |  $k = 2-10$  Groups | Within-Group  $df = 1-\infty$  | For Tukey's HSD Post-Hoc Test After ANOVA

**Source:** StatisticsFundamentals.com | statisticsfundamentals.com/tables/tukeys-q-table/ | **Derived from:** Tukey (1949), Biometrics 5(2):99-114 | NIST/SEMATECH e-Handbook (itl.nist.gov) | Penn State STAT 503 (online.stat.psu.edu) | Kutner et al. (2005), Applied Linear Statistical Models, 5th ed. | Abdi & Williams (2010), Encyclopedia of Research Design, SAGE | FDA Multiple Endpoints Guidance (fda.gov)

$HSD = q(\alpha, k, df) \times \sqrt{(MSE / n)}$  |  $df_{within} = N - k$  | **Reject  $H_0$  for pair  $(i, j)$  when  $|\bar{y}_i - \bar{y}_j| > HSD$**  | **Tukey-Kramer (unequal n):  $SE = \sqrt{(MSE/2 \times (1/n_1 + 1/n_2))}$**

**Table 1 — Studentized Range Critical Values,  $\alpha = 0.05$  (95% Confidence)**

df \ k	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9	k=10
1	17.97	26.98	32.82	37.08	40.41	43.12	45.40	47.36	49.07
2	6.08	8.33	9.80	10.88	11.74	12.44	13.03	13.54	13.99
3	4.50	5.91	6.82	7.50	8.04	8.48	8.85	9.18	9.46
4	3.93	5.04	5.76	6.29	6.71	7.05	7.35	7.60	7.83
5	3.64	4.60	5.22	5.67	6.03	6.33	6.58	6.80	6.99
6	3.46	4.34	4.90	5.30	5.63	5.90	6.12	6.32	6.49
7	3.34	4.16	4.68	5.06	5.36	5.61	5.82	6.00	6.16
8	3.26	4.04	4.53	4.89	5.17	5.40	5.60	5.77	5.92
9	3.20	3.95	4.41	4.76	5.02	5.24	5.43	5.59	5.74
10	3.15	3.88	4.33	4.65	4.91	5.12	5.30	5.46	5.60
11	3.11	3.82	4.26	4.57	4.82	5.03	5.20	5.35	5.49
12	3.08	3.77	4.20	4.51	4.75	4.95	5.12	5.27	5.39
13	3.06	3.73	4.15	4.45	4.69	4.88	5.05	5.19	5.32
14	3.03	3.70	4.11	4.41	4.64	4.83	4.99	5.13	5.25
15	3.01	3.67	4.08	4.37	4.60	4.78	4.94	5.08	5.20
16	3.00	3.65	4.05	4.33	4.56	4.74	4.90	5.03	5.15
17	2.98	3.63	4.02	4.30	4.52	4.70	4.86	4.99	5.11
18	2.97	3.61	4.00	4.28	4.49	4.67	4.82	4.96	5.07
19	2.96	3.59	3.98	4.25	4.47	4.65	4.79	4.92	5.04
20	2.95	3.58	3.96	4.23	4.45	4.62	4.77	4.90	5.01
24	2.92	3.53	3.90	4.17	4.37	4.54	4.68	4.81	4.92

df \ k	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9	k=10
30	2.89	3.49	3.85	4.10	4.30	4.46	4.60	4.72	4.82
40	2.86	3.44	3.79	4.04	4.23	4.39	4.52	4.63	4.73
60	2.83	3.40	3.74	3.98	4.16	4.31	4.44	4.55	4.65
120	2.80	3.36	3.68	3.92	4.10	4.24	4.36	4.47	4.56
∞	2.77	3.31	3.63	3.86	4.03	4.17	4.29	4.39	4.47

■ Standard choice for most published research.  $q$  values increase with  $k$ ; use  $df = N - k$  from your ANOVA within-groups row.

**Table 2 — Studentized Range Critical Values,  $\alpha = 0.01$  (99% Confidence)**

df \ k	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9	k=10
1	90.03	135.00	164.30	185.60	202.20	215.80	227.30	237.00	245.60
2	14.04	19.02	22.29	24.72	26.63	28.20	29.53	30.68	31.69
3	8.26	10.62	12.17	13.33	14.24	15.00	15.64	16.20	16.69
4	6.51	8.12	9.17	9.96	10.58	11.10	11.55	11.93	12.27
5	5.70	6.98	7.80	8.42	8.91	9.32	9.67	9.97	10.24
6	5.24	6.33	7.03	7.56	7.97	8.32	8.61	8.87	9.10
7	4.95	5.92	6.54	7.01	7.37	7.68	7.94	8.17	8.37
8	4.75	5.64	6.20	6.62	6.96	7.24	7.47	7.68	7.86
9	4.60	5.43	5.96	6.35	6.66	6.91	7.13	7.33	7.49
10	4.48	5.27	5.77	6.14	6.43	6.67	6.87	7.05	7.21
11	4.39	5.15	5.62	5.97	6.25	6.48	6.67	6.84	6.99
12	4.32	5.05	5.50	5.84	6.10	6.32	6.51	6.67	6.81
13	4.26	4.96	5.40	5.73	5.98	6.19	6.37	6.53	6.67
14	4.21	4.89	5.32	5.63	5.88	6.08	6.26	6.41	6.54
15	4.17	4.84	5.25	5.56	5.80	5.99	6.16	6.31	6.44
16	4.13	4.79	5.19	5.49	5.72	5.92	6.08	6.22	6.35
17	4.10	4.74	5.14	5.43	5.66	5.85	6.01	6.15	6.27
18	4.07	4.70	5.09	5.38	5.60	5.79	5.94	6.08	6.20
19	4.05	4.67	5.05	5.33	5.55	5.73	5.89	6.02	6.14
20	4.02	4.64	5.02	5.29	5.51	5.69	5.84	5.97	6.09

df \ k	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9	k=10
24	3.96	4.55	4.91	5.17	5.37	5.54	5.69	5.81	5.92
30	3.89	4.45	4.80	5.05	5.24	5.40	5.54	5.65	5.76
40	3.82	4.37	4.70	4.93	5.11	5.26	5.39	5.50	5.60
60	3.76	4.28	4.59	4.82	4.99	5.13	5.25	5.36	5.45
120	3.70	4.20	4.50	4.71	4.87	5.01	5.12	5.21	5.30
$\infty$	3.64	4.12	4.40	4.60	4.76	4.88	4.99	5.08	5.16

■ Use for clinical trials, pharmaceutical dose comparisons, or confirmatory research where false positives carry high consequences.

# Tukey's Q Table — Complete Reference Card ■ Page 2: How to Use & Worked Example

## How to Read Tukey's Q Table

**Step 1: Run one-way ANOVA.** Confirm the F-test is significant. Record MSE and  $df = N - k$  from the within-groups row.

**Step 2: Count k.** The total number of groups/treatments in the study, including any control.

**Step 3: Read df from ANOVA.** Within-group  $df = N - k$ . Find this row in the left column.

**Step 4: Choose alpha.** Use  $\alpha = 0.05$  (Table 1) for standard work;  $\alpha = 0.01$  (Table 2) for stringent testing.

**Step 5: Intersect k and df.** The cell at their crossing is  $q(\alpha, k, df)$ .

**Step 6: Compute HSD.**  $HSD = q * \sqrt{MSE / n}$ . Any pair with  $|y_i - y_j| > HSD$  is significant.

## Key Formulas

$q(\alpha, k, df) \times \sqrt{MSE / n}$  [HSD]

$(X_{max} - X_{min}) / \sqrt{MSE/n}$  [q statistic]

$N - k$  [df within]

$\sqrt{MSE/2 \times (1/n_1 + 1/n_2)}$  [Tukey-Kramer SE]

controlled at alpha [FWER]

## Worked Example (k=4, df=20, alpha=0.05)

Scenario: An agricultural researcher tests  $k = 4$  soy crop configurations, each with  $n = 6$  plots ( $N = 24$ ). One-way ANOVA returns  $F(3,20) = 6.18^*$ ,  $MSE = 11.88$ . Tukey's HSD identifies which configurations differ.

Source	df	MS	F
Between	3	73.50	6.18*
Within	20	11.88 (MSE)	
Total	23		

**Step 1:** From  $\alpha=0.05$  table,  $k=4$  col,  $df=20$  row  $\rightarrow q = 3.96$

**Step 2:**  $HSD = 3.96 \times \sqrt{11.88/6} = 3.96 \times 1.407 = 5.57$

**Step 3:** Group means:  $A=12.1, B=18.0, C=11.8, D=14.5$

Pair	$ y_i - y_j $	vs $HSD=5.57$	Result
A vs B	5.90	$> 5.57$	SIG *
A vs C	0.30	$< 5.57$	n.s.
A vs D	2.40	$< 5.57$	n.s.
B vs C	6.20	$> 5.57$	SIG *
B vs D	3.50	$< 5.57$	n.s.
C vs D	2.70	$< 5.57$	n.s.

*B differs significantly from A and C at  $\alpha=0.05$ . FWER is controlled at 5% across all 6 comparisons.*

## Post-Hoc Method Selection

Method	Comparisons	FWER	Best Power For	When to Use
Tukey's HSD	All pairs	Yes	All-pairs testing	Compare every group to every other
Bonferroni	Pre-specified	Yes	Few planned tests	Small subset of planned comparisons
Dunnett's	Each vs control	Yes	Control comparisons	Each treatment vs one control group
Scheffe's	All contrasts	Yes	Complex contrasts	Arbitrary linear combinations of means