Poisson Distribution — Exam Reference Card

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Formula & Key Notation

Symbol / Notation	Formula / Value	Meaning
$P(X = k)$	$e^{-\lambda} \times \lambda^k / k!$	Probability of exactly k events (PMF)
$P(X \leq k)$	$\Sigma P(X=i)$ for $i = 0$ to k	Probability of k or fewer events (CDF)
$P(X > k)$	$1 - P(X \leq k)$	Complement: probability of more than k events
$P(X \geq k)$	$1 - P(X \leq k-1)$	Probability of at least k events
Mean $E(X)$	λ	Expected number of events per interval
Variance $Var(X)$	λ	Variance equals mean — unique to Poisson
SD σ	$\sqrt{\lambda}$	Standard deviation
Skewness	$1 / \sqrt{\lambda}$	Right-skewed for small λ; near-symmetric for $\lambda > 5$

The 4 Conditions for Valid Poisson Modeling

1. Independence
Events do not influence each other.

2. Constant rate
λ does not change across the interval.

3. No simultaneity
Two events cannot occur at the same instant.

4. Proportionality
P(event) ∝ interval length.

P(X = k) — PMF Quick Reference Table

Most-used exam values. Highlighted row: $k = \lambda$ (approximate mode).

k	λ=0.5	λ=1.0	λ=1.5	λ=2.0	λ=2.5	λ=3.0	λ=4.0	λ=5.0
0	**0.6065**	0.3679	0.2231	0.1353	0.0821	0.0498	0.0183	0.0067
1	0.3033	**0.3679**	**0.3347**	0.2707	0.2052	0.1494	0.0733	0.0337
2	0.0758	0.1839	0.2510	**0.2707**	**0.2565**	0.2240	0.1465	0.0842
3	0.0126	0.0613	0.1255	0.1804	0.2138	**0.2240**	0.1954	0.1404
4	0.0016	0.0153	0.0471	0.0902	0.1336	0.1680	**0.1954**	0.1755
5	0.0002	0.0031	0.0141	0.0361	0.0668	0.1008	0.1563	**0.1755**
6	0.0000	0.0005	0.0035	0.0120	0.0278	0.0504	0.1042	0.1462
7	0.0000	0.0001	0.0008	0.0034	0.0099	0.0216	0.0595	0.1044
8	0.0000	0.0000	0.0001	0.0009	0.0031	0.0081	0.0298	0.0653
9	0.0000	0.0000	0.0000	0.0002	0.0009	0.0027	0.0132	0.0363
10	0.0000	0.0000	0.0000	0.0000	0.0002	0.0008	0.0053	0.0181
11	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0019	0.0082

Purple-highlighted cells show the approximate mode ($k \approx \lambda$ — highest probability value in each column). For the complete table covering λ up to 10.0 and k up to 15, see the PMF Table PDF.

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P(X ≤ k) — CDF Quick Reference Table

Use for "at most k", "no more than k", and "fewer than k+1" questions.

k	λ=0.5	λ=1.0	λ=1.5	λ=2.0	λ=2.5	λ=3.0	λ=4.0	λ=5.0
0	0.6065	0.3679	0.2231	0.1353	0.0821	0.0498	0.0183	0.0067
1	0.9098	0.7358	0.5578	0.4060	0.2873	0.1991	0.0916	0.0404
2	0.9856	0.9197	0.8088	0.6767	0.5438	0.4232	0.2381	0.1247
3	0.9982	0.9810	0.9344	0.8571	0.7576	0.6472	0.4335	0.2650
4	0.9998	0.9963	0.9814	0.9473	0.8912	0.8153	0.6288	0.4405
5	1.0000	0.9994	0.9955	0.9834	0.9580	0.9161	0.7851	0.6160
6	1.0000	0.9999	0.9991	0.9955	0.9858	0.9665	0.8893	0.7622
7	1.0000	1.0000	0.9998	0.9989	0.9958	0.9881	0.9489	0.8666
8	1.0000	1.0000	1.0000	0.9998	0.9989	0.9962	0.9786	0.9319
9	1.0000	1.0000	1.0000	1.0000	0.9997	0.9989	0.9919	0.9682
10	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9972	0.9863
11	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9991	0.9945

Green values = 1.0000 (all probability mass accounted for at this k). Complement: P(X > k) = 1 – P(X ≤ k). "At least k": P(X ≥ k) = 1 – P(X ≤ k–1).

Worked Examples

Ex 1 — Exact probability	Call centre: λ = 3 calls/hr. Find P(X = 5).	PMF table: λ=3.0, k=5 → **0.1008**	**10.08% chance of exactly 5 calls.**
Ex 2 — Cumulative	Server: λ = 2 requests/sec. Find P(X ≤ 3).	CDF table: λ=2.0, k=3 → **0.8571**	**85.71% probability of 3 or fewer requests.**
Ex 3 — Complement	Factory: λ = 4 defects/roll. Find P(X > 6).	1 – P(X ≤ 6) = 1 – 0.8893 → **0.1107**	**11.07% probability of more than 6 defects.**
Ex 4 — Interval scaling	Bacteria: 1.5 colonies/cm². Find P(X < 3) in 2 cm².	Scale: λ = 1.5 × 2 = 3.0. P(X ≤ 2): CDF → **0.4232**	**42.32%. Always adjust λ to match the interval.**

Poisson vs Binomial — Approximation Rule

Poisson	λ (rate)	λ	λ	Rare event counts over intervals	Approx. Binomial when n≥20, p≤0.05, λ=np
Binomial	n, p	np	np(1–p)	Fixed yes/no trials	Use Normal when n large, p not extreme

Common Exam Mistakes

✗ Using λ without scaling to the interval
→ Always multiply λ by the interval ratio. Rate 3/hour over 2 hours → λ = 6.

✗ Using PMF when CDF is needed
→ "At most k" and "no more than k" always require the CDF table.

✗ Forgetting the complement rule

→ $P(X > k) = 1 – P(X \leq k)$, not $1 – P(X = k)$.

✗ Applying Poisson when events cluster

→ Clustered events violate independence. Use negative binomial instead.